

Method V : Exact diff. eqⁿ:

A eqⁿ. of the form $M(x, y) dx + N(x, y) dy = 0$ is exact
iff $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ ①

Working Rule: If ① is exact then sol. is given by

$$(I) \quad \int M \, dx + \int N \, dy = C$$

keep y constant keep only those terms that do not contain x

this method sometimes doesn't give complete sol.

$$(II) \quad A = \int M \, dx \quad B = \int N \, dy$$

keep y constant keep x constant

$A + B = C$
As it is leave those terms that appear in A.

Eg. 1) $y \sin 2x \, dx - (y^2 + \cos^2 x) \, dy = 0$

$\underbrace{y \sin 2x}_M \quad \underbrace{-(y^2 + \cos^2 x)}_N$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} (y \sin 2x) = \sin 2x \quad \frac{\partial}{\partial y} y = \sin 2x$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} (-y^2 - \cos^2 x) = 0 - 2 \cos x (-\sin x) = \sin 2x$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{exact diff. eqⁿ}. \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$(I) \quad \int y \sin 2x \, dx + \int -y^2 \, dy = C$$

y constant

$$y \int \sin 2x \, dx - \frac{y^3}{3} = C$$

$$- \frac{y \cos 2x}{2} - \frac{y^3}{3} = C \quad \underline{\text{Ans.}}$$

$$\int y \sin 2x \, dx = - \frac{y \cos 2x}{2}$$

$$\text{II) } A = \int_{y \text{ constant}}^2 M dx = \int_{y \text{ constant}}^3 y \sin 2x dx = -y \frac{\cos 2x}{2}$$

$$B = \int_{x \text{ constant}} N dy = \int (-y^2 - \cos^2 x) dy$$

$$= -\frac{y^3}{3} - y \cos^2 x = -\frac{y^3}{3} - y \left(\frac{1 + \cos 2x}{2} \right)$$

$$A + B = K$$

$$-\frac{y \cos 2x}{2} - \frac{y^3}{3} - \frac{y}{2} = K$$

Ans.

$$\text{Eg. 2) } \underbrace{(2x \sin y + y^3 e^x)}_M dx + \underbrace{(x^2 \cos y + 3y^2 e^x)}_N dy = 0$$

$$\frac{\partial M}{\partial y} = 2x \cos y + 3y^2 e^x$$

$$\frac{\partial N}{\partial x} = 2x \cos y + 3y^2 e^x$$

} eq. is exact.

$$A = \int_{y \text{ constant}} M dx = \int_{y \text{ const}} (2x \sin y + y^3 e^x) dx$$

$$= x^2 \sin y + y^3 e^x$$

$$B = \int_{x \text{ const}} N dy = \int_{x \text{ const}} (x^2 \cos y + 3y^2 e^x) dy = x^2 \sin y + e^x y^3$$

$$\text{Sol. } A + B = K$$

$$x^2 \sin y + y^3 e^x = K$$

Ans.

$$\text{Eg. } \underbrace{y dx}_M + \underbrace{x dy}_N = 0$$